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A STUDY ON THE TIME VARYING OPTIMAL CONTROL OF LINEAR DIFFERENTIAL ALGEBRAIC EQUATIONS USING RATIONAL APPROXIMATION METHOD

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Abstract

This paper deals with the implementation of rational approximation method to solve the optimal control problem of linear time varying singular systems with a quadratic cost functional. The idea is that the states and the input are expressed in terms of RAM. The method simplifies the system of state equations into a set of algebraic equations which can be solved using a digital computer. Illustrative example is included to demonstrate the validity and applicability of the technique.

Mathematics Subject Classification: 65L05, 65L80.Key Words and Phrases: Optimal control, Singular Systems, Pade Approximation, Rational Approximation Method.

1 Introduction

Many industrial processes, such as those in the steel and oil industries, need optimal management of unique systems. The issue of optimal control of singular systems has attracted a lot of attention, particularly from computational mathematicians who are studying control theory issues and trying to figure out how to compute the value of the control vector that governs the state vector.

Many electrical circuits mechanical multi-body systems need the use of mathematical models optimum singular systems. They are especially helpful when a nonlinear system that is linear along a nominal trajectory is subject to algebraic limitations. Theories that are well-established for regular systems without generalization may not apply to these systems because their behaviors may differ greatly from those of regular linear systems. Given this context, over the last thirty years, a large number of research publications have focused on solitary systems.

In many control issues, determining the best control subject to linear quadratic cost is essential. However, the quadratic optimum control issue for unique systems for linear systems in regular state-space was not studied until the 1980s. Cobb [6] is one of two well-known papers that addresses this problem in the time-variant environment. By Wansheng Tang et al. [11], the time-optimal control of singular systems was examined.

Using Walsh series, Chen and Hsiao [4], Chen and Shih [5], investigated the issue of best for linear systems that are both time-invariant and time-varying. Mentioned that Cobb [6] and Pandolfi [10] appear to have been the first writers to examine the optimum regulator problem of continuous time singleton systems, earlier research. They both employed state feed-backs, and Ricatti-type matrix equations helped them reach their conclusions.

The STWS approach was first used to the study and optimum control of linear systems by Palanisamy [9]. The STWS approach has been used by Balachandran and Murugesan [1]-[3] for the study of electronic circuits, non-linear singular systems, optimum control of singular systems, and many types of first order systems. This chapter presents the rational approximation technique for studying the best control for time-varying linear singular systems. In this included chapter, we introduce the rational approximation method to study the optimal control of linear singular systems (time varying).

2 Optimal control of singular systems

The linear time–variant singular system represented in [2] is considered. Assuming that

$$\det(s - K - A)\ 6\neq 0,\ B = \begin{bmatrix} 0 \\ Ip \end{bmatrix},\ K = \begin{bmatrix} K1 \\ k2 \end{bmatrix}$$

Where $K_1 = \begin{bmatrix} I_{n-p} \\ 0 \end{bmatrix}$. Now the problem can be stated as follows: Given the initial state

$x(0-) = x_0$ find a control vector u(t) that generates a state x(t) such that $x(t_f) = x_f$, where t_f is a prescribed time and x_f is a fixed vector, and minimizes the cost functional

$$J = \int_0^{t_f} L(x,u)dt$$

where Q and R stand for the specified real symmetric constant matrices, and L = 1 (xT Qx + uT Ru). The approach of E1–Tohami et al. [7] may be used to rebuild the state if the starting state x (0—) is unknown. Lovass-Nagy et al. [8] have demonstrated that the challenge of determining an ideal control simplifies to the resolution of a singular systems.

$$\text{Let } x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

x_1 is (n - p) × 1 and x_2 is p × 1, $K_2 = \begin{bmatrix} K_{21} \\ K_{22} \end{bmatrix}^T$ where K_{21} is p× (n – p) and K_{22} is $p\ x\ p$

$$A = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix},\ A_1 = \begin{bmatrix} A_{11} \\ A_{12} \end{bmatrix}^T,\ A_2 = \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix}^T$$

where A_{11},A_{12},A_{21},A_{22} are respectively *(n-p) × (n-p), (n-p) ×p, p× (n-p), p×p*. Further take

Q $=\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}$ where Q1 and Q2 are *(n - p)* ×*n* and *p*×*n* respectively. Then we have the following equations (Lovass-Nagy et al. [8])

$$\frac{dx}{dt} = A_{11}x_1 + A_{12}x_2 \qquad (1)$$

$$K_{21}\frac{dx_1}{dx} + K_{22}\frac{dx_1}{dx} = A_{21}x_1 - A_{22}x_2 = u \qquad (2)$$

$$\frac{dp_1}{dt} = - A^T_{11}p_1 + K^T_{21}R\frac{du}{dt} + A^T_{21}Ru - Q_1x \qquad (3)$$

$$A^T_{12}p_1 = K^T_{22}R\frac{du}{dt} + A^T_{22}Ru - Q_2x \qquad (4)$$

where p = $\begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$ is the co-state vector corresponding to time varying singular systems. From the equations (1)-(4), optimal state and optimal control can be calculated.

The previously indicated set of equations (1) to (4) may be used to generate the governing equations for u(t) and x(t) for the time-invariant and time-varying optimal control problem. It should be remembered that not all time-invariant optimal control problems can be solved using the governing equations mentioned above. In order to obtain the governing equations entirely (a generalized form) for the time-invariant optimal control problem of the type in [2], more research must be done.

3 Formulation of time–varying optimal control of singular systems

Rearranging Equations (1-4), we have the following system.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ K_{21} & K_{22} & 0 & 0 \\ 0 & 0 & K^T_{22} & 0 \\ 0 & 0 & -K^T_{21} & 1 \end{bmatrix} \begin{bmatrix} x'_1 \\ x'_2 \\ u' \\ p' \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & 1 & 0 \\ Q^T_{12} & Q_{22} & -A_{22}R & A^T_{12} \\ -Q_{11} & -Q_{12} & A^T_{21} & -A^T_{11} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u \\ p_1 \end{bmatrix}$$

which can be written in the form

$$K(t)y'(t) = M(t)y(t)$$

Where,

$$K(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ K_{21} & K_{22} & 0 & 0 \\ 0 & 0 & K_{22}^T & 0 \\ 0 & 0 & -K_{21}^T & 1 \end{bmatrix}$$

$$M(t) = \begin{bmatrix} A_{11} & A_{12} & 0 & 0 \\ A_{21} & A_{22} & 1 & 0 \\ Q_{12}^T & Q_{22} & -A_{22}R & A_{12}^T \\ -Q_{11} & -Q_{12} & A_{21}^T & -A_{11}^T \end{bmatrix}$$

and y = x1 x2 u p1 where the matrix K(t) is unique and some of the components are time dependent. As a result, it cannot be expressed in the conventional form and is referred to as a time-varying "singular system," "descriptor system," or "generalized state space system."

4 Example for the time–varying optimal control of singular systems

It is thought that the linear singular system [6, 8]

$$\begin{bmatrix} 1 & 0 \\ -t & 0 \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1+t & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad (5)$$

with initial condition x(0) = $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and the performance index

$$J = \frac{1}{2} \int_0^{t_f} (x^T x + u^2) dt$$

The goal is to minimize the above cost functional (performance index) and find the best control u(t) that will move the system from an acceptable beginning state x(0—) = x0 to a desirable end state xf in a specified amount of time t_f.

The time-varying optimum control of singular systems has an accurate solution that is

$$x_1(t) = exp(—t)$$
$$x_2(t) = exp(—t) + sin(t) \quad (6)$$

and the optimal control is

$$u(t) = sin(t) \quad (7)$$

The discrete time solutions and precise solutions of z(t) are calculated using PAM, RAM method, and the following equations (5)-(7). These are provided in Tables 1 and 2 together with the solution found by Balachandran and Murugesan [2] using Single Term Walsh Series approach. The findings are shown in Tables 1 and 2. The associated optimum control, u(t), is computed using the PAM, RAM technique, and the aforementioned equations (5)– (7).

Table 1: Discrete solutions for $z_1(t)$ and $z_2(t)$

	Exact Solution		PAM Solution		RAM Solution	
t	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$	$x_1(t)$	$x_2(t)$
0.00	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.25	0.778801	1.026204	0.778801	1.026204	0.778801	1.026204
0.50	0.606531	1.085956	0.606531	1.085956	0.606531	1.085956
0.75	0.472367	1.154005	0.472367	1.154005	0.472367	1.154005
1.00	0.367879	1.209350	0.367879	1.209350	0.367879	1.209350
1.25	0.286504	1.235489	0.286504	1.235489	0.286504	1.235489
1.50	0.223130	1.220625	0.223130	1.220625	0.223130	1.220625
1.75	0.173774	1.157759	0.173774	1.157759	0.173774	1.157759
2.00	0.135335	1.044632	0.135335	1.044632	0.135335	1.044632

5 Conclusions

The rational approximation approach is effective in determining the state vector and the control input vector, as demonstrated by the obtained results of the time-varying optimum control of linear singular systems with quadratic performance index. Tables 1 – 2 show that, for the majority of time intervals, the rational approximation approach produces a lower absolute error than the Pad´e approximation method, which yields both a small error and the exact answers to the issue. Based on the findings displayed in Figures 1 and 2, it can be concluded that, in solving the time-variant optimum control of linear singular systems with quadratic fluctuations, the error in the rational approximation approach is much smaller than in the Pade approximation technique.

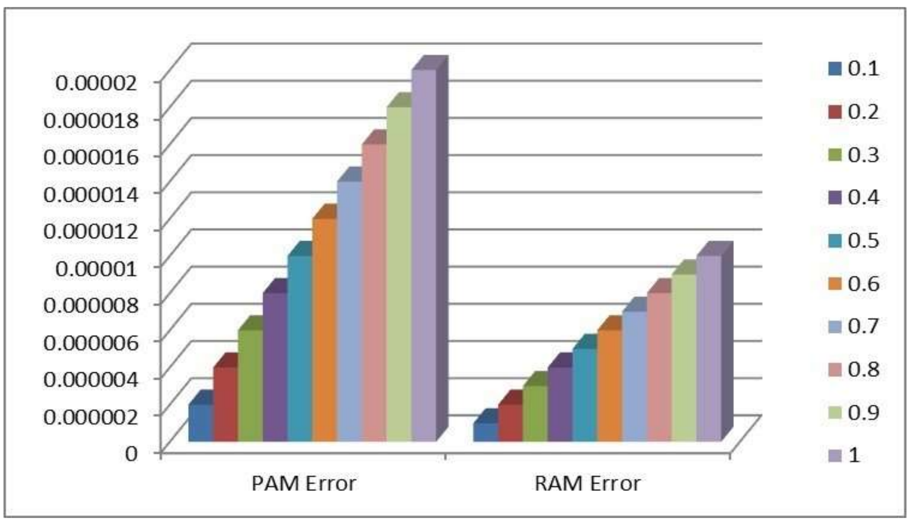

Figure 1: Error graph for the state $x_1(t)$

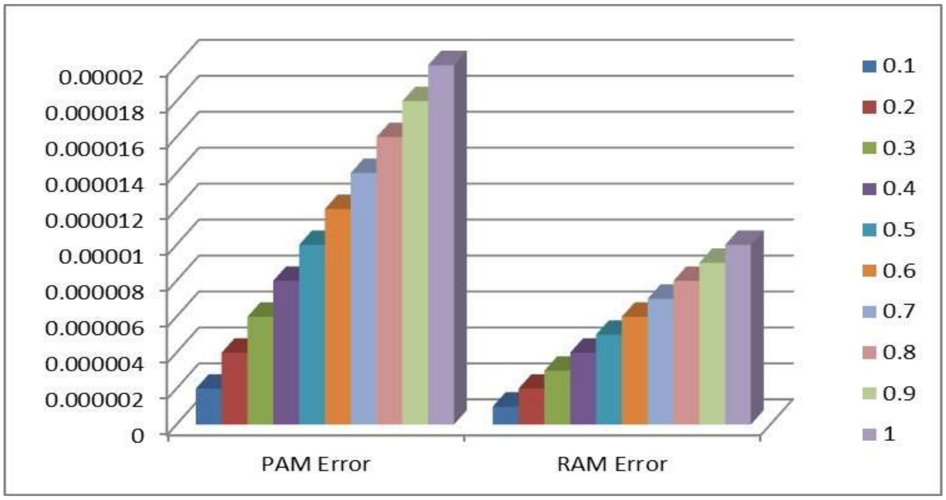

Figure 2: Error graph for the state $x_2(t)$

Table 2: Discrete solutions for $u(t)$

t	Exact Solution $u(t)$	PAM Solution $u(t)$	RAM Solution $u(t)$
0.00	0.000000	0.000000	0
0.25	0.247404	0.247404	0.247404
0.50	0.479426	0.479426	0.479426
0.75	0.681639	0.681639	0.681639
1.00	0.841471	0.841471	0.841471
1.25	0.948984	0.948984	0.948984
1.50	0.997495	0.997495	0.997495
1.75	0.983986	0.983986	0.983986
2.00	0.909297	0.909297	0.909297

The rational approximation approach outperforms the Pade approximation technique for solving the time-varying optimum control of linear singular systems with quadratic performance index, according to the error graphs shown in Figures 1–2. The rational approximation approach has a very high accuracy when compared to the RK method when beginning step-size, calculation time, and error from Tables 1–3 are taken into account. Therefore, the rational approximation technique is more suited for researching time-varying solitary system optimum control problems.

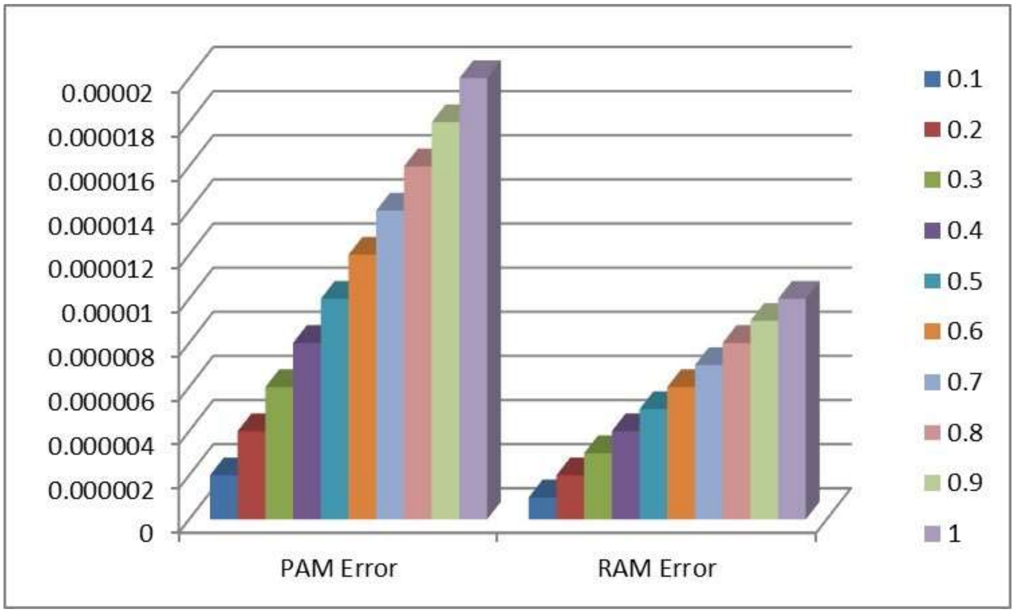

Figure 3: Error graph for the state $x_3(t)$

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